DEPARTMENT OF MATHEMATICS

Application of Linear Algebra to Networks (Graph Theory)

By

Group 7

March, 2026

AN ESSAY TO THE DEPARTMENT OF MATHEMATICS, UNIVERSITY OF GHANA, AS A REQUIREMENT FOR MATH 351 (LINEAR ALGEBRA) COURSE.

Group Members

- Amoah Kwame Adjei (22033203)
- Adwoa Pokua Aboagye (22235227)
- Sintim Kofi Adu (11354716)
- Ishmael Affum Kwakye (22032451)
- Melchizedek Sensemore (22234596)

Supervisor: Ativor Svetlana

Signature:

Date: 30th March 2026

ACKNOWLEDGEMENTS

We would like to thank our lecturer for the course, MATH 351: Linear Algebra, Dr, Kenneth Dadedzi, for providing us a platform to undertake this research. Our gratitude is also extended to our project supervisor Svetlana, for her unwavering and useful support in the creation of this work.

DEDICATION

We dedicate this work to the Department of Mathematics and all its faculty,for teaching us how we can apply what we learn in the class to real life and to solve actual problems.

Abstract

As students, we have noticed the lack of stable and strong WiFi on the University of Ghana Campus, and we decided to analyze this problem within the context of graph theory and linear algebra. The main aim of this paper is to understand how a specific network deficient area could be catered for by increasing the overall connectivity across halls, hostels, and buildings labelled as nodes, and the distance between them as edges, and applying matrix algebra and a greedy algorithm to approximate locations at which to place new WiFi access points.

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Chapter 1

Introduction

1.1 Introduction

1.1.1 Project Scope

This project mainly seeks to explore the relationship between linear algebra and graph theory by applying it to a real life scenario, analysing the results, and determining how both fields interesect with each other. We would be mainly dealing with concepts such as matrices, graphs, and eigenvalues, core principles required in any work concerning graph theory and linear algebra.

1.1.2 Literature Review

Graph theory has long been used in modelling relationships between parameters and variables. Many phenomena in nature can be represented as graphs, and the operations performed on these graphs can be defined using matrix algebra, which is where linear algebra comes in. Some applications of graph theory include GPS and route planning , social networks and disease spread, google search and website ranking etc. whilst linear algebra provides the solid framework required to explore graphs discretely and computationally. The foundational concepts for this work include eigenvalues, the linear independence of vector spaces, and span. These provide the requisite foundation for the construction of not only the graphs themselves, but for the algorithms used to calculate optimal paths. Eigenvalues of graph matrices encapsulate global properties like connectivity and stability. Linear independence ensures that the representation of paths or states is unique and non-redundant, while the concept of span guarantees that our search and computational methods remain within the defined vector space of possible solutions, preventing logical excursions beyond the problem's search space.

1.1.3 Statement of the research

The main aim of this paper is to analyze the optimal placement of WiFi access points(APs) on the University of Ghana campus, in order to increase the number of edges connecting any given AP and a potential internet connection spot. We make extensive use of different types of matrices, eigenvalues, and certain algorithms.

1.1.4 Structure of the work

- Chapter 1 : In this chapter, we will define some basic concepts related to both graph theory and linear algebra including graphs, connectivity and the Laplacian matrix.
- Chapter 2 : We define our research methodology, data collection methods, and analysis of said data in this chapter.
- Chapter 3 : Here, we describe a specific use case scenario based on the data collected and explore our results. We also extend this to a more generalized scenario for any n number of access points we want to add to a network.
- Chapter 4 : We discuss our findings and conclusions in this chapter.
- Chapter 5 : We conclude our paper and provide a summary of all the work and finding we have made throughout.

Chapter 2

Basic Notion

2.1 Introduction

In this chapter, we introduce some key concepts and terms we would regularly be using in our work.

2.2 Key Concepts and Defintions

Graph theory and networks are used almost everywhere once you start looking for relationships between things, we can analyze graphs using key concepts in linear algebra as well.

2.3 What is Graph Theory?

Graph theory seeks to explore the structure and specifics of graphs, how they can be applied in a variety of problems, and the mathematics that can be done with them. The structure of graphs can be analyzed using matrices, which is the main intersection of **Graph Theory** and **Linear Algebra**.

2.3.1 Graph Representation and Domain Specific Matrices Used

Definition 2.3.2. A **graph** $G = (V, E)$ consists of a set V of Vertices (nodes) and a set E of edges (connections or relationships).

Definition 2.3.3. A **bipartite graph** B is a graph whose vertices can be divided into two disjoint and independent sets, U and V such that every edge connects a vertex in U to one in V. Formally, a graph $G = (V, E)$ is bipartite if there exists a partition of the vertex V into two sets V_1 and V_2 such that:

- $V_1 \cup V_2 = V$
- $V_1 \cap V_2 = \emptyset$

Any edge e can be represented as a distinct pair of vertices. For instance: $[2,3]$. Based on the graph structure, we define the following:

- **Vertices:** (v_1, v_2, v_3, v_4)
- **Edges:** $[1,2], [2,3], [1,4], [2,4]$

Definition 2.3.4 (Matrix)**.** A matrix is a rectangular array.

Definition 2.3.5 (The Adjacency Matrix (A))**.** Adjacency of a matrix is represented as A. For a graph G with n vertices, the Adjacency Matrix A is defined where

$$A_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.3.6 (The Laplacian Matrix (L))**.** The Laplacian Matrix (L) is an $n \times n$ symmetric matrix and serves as an excellent tool for modeling the behavior of a network.

It is defined as the difference between the Degree Matrix (D) and the Adjacency Matrix (A):

$$L = D - A \tag{2.3.1}$$

Where D is the diagonal matrix consisting of node degrees ($D_{ii} = d_i$).

2.3.7 Key Characteristics of the Laplacian

1. The sum of the rows and columns of L always adds up to zero, making it a singular matrix.
2. L is symmetric and positive semi-definite, meaning all its eigenvalues are real and non-negative: $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$.

2.4 Spectral Properties and Connectivity

The Spectrum (eigenvalues) of the Laplacian reveals the underlying structure of a network.

2.4.1 Connectivity Analysis

1. The smallest eigenvalue λ_1 is always 0, with a corresponding eigenvector consisting of all 1s.
2. The algebraic multiplicity of the eigenvalue 0 (the number of times it appears) is equal to the number of connected components in the graph.
3. The second smallest eigenvalue, λ_2, is known as the **Fiedler value.** We would be making extensive use of this value within our algebraic connectivity analysis.
4. A network is considered connected if and only if $\lambda_2 > 0$.

Chapter 3

Data Collection and Implementation

3.1 Introduction

We explore the use of all the matrices defined in the previous chapter and implement our research within a small scale scenario.

3.2 Key Research Problem

As students, we have noticed the lack of stable and strong WiFi on the University of Ghana Campus, and we decided to analyze this problem within the context of graph theory and linear algebra. The main aim of this paper is to understand how a specific network deficient area could be catered for by increasing the overall connectivity across halls, hostels, and buildings labelled as nodes, and the distance between them as edges, and applying matrix algebra and a greedy algorithm to approximate locations at which to place new WiFi access points.

3.2.1 Research Methodology and Information Statistics

At the beginning of this project, we sought opinions of students and sent out a google form so we could get a fair idea of where the key problem areas were. Approximately 33.3% of students connected to the school WiFi from their hostels or halls, a close runner up to that was the 28.2% of students who connected mainly from the Balme Library. More interestingly, many students cited locations such as lecture halls (JQB, New N Block, N Block, etc.) as places where WiFi connection was hard to come by, especially within certain departments. This information helped us to consider a small patch of the university campus which we refer to as our “toy model”, where we could analyze connectivity within an isolated area where an existing WiFi access point provides enough coverage in itself, but does not cater for nearby buildings.

3.2.2 Analysis Workflow

Within our research, we converted a part of the campus into a bipartite graph network, with our disjoint sets being halls and/or places to connect to WiFi, and WiFi access points (subsequently referred to as AP). From our survey, our specific use case utilized the Balme Library as the origin from which all our other nodes are situated, due to the maximum signal strength from it.

3.3 Numerical Scenario for a Fragment of UG Campus

We consider a simplified campus fragment consisting of three buildings and two existing access points (APs), with a new access point $\mathbf{AP}_C$.

Node	Description	Coordinates
B_1	Library	$(0,\ 0)$
B_2	JQB	$(100,\ 0)$
B_3	Hostel	$(50,\ 87)$
AP_A	Existing access point	$(25,\ 0)$
AP_B	Existing access point	$(75,\ 43)$
AP_C	Candidate access point	$(50,\ 0)$

Table 3.1: Node coordinates for a fragment of UG campus model.

3.4 Graph Structure

The campus is modelled as a weighted bipartite graph $G = (V, E)$ where buildings form one partition and APs the other. Edge weights are signal strengths computed via the inverse square law:

$$\text{signal} = \frac{S_0}{d^2}, \qquad S_0 = 10{,}000$$

where d is the Euclidean distance between building and AP calculated using the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

3.4.1 Laplacian Matrix Before Adding $\mathbf{AP_C}$

With nodes ordered as $(B_1,\ B_2,\ B_3,\ \text{AP}_A,\ \text{AP}_B)$, the adjacency matrix is:

$$A = \begin{pmatrix} 0 & 0 & 0 & 16.00 & 1.35 \\ 0 & 0 & 0 & 1.78 & 5.44 \\ 0 & 0 & 0 & 1.21 & 5.17 \\ 16.00 & 1.78 & 1.21 & 0 & 0 \\ 1.35 & 5.44 & 5.17 & 0 & 0 \end{pmatrix}$$

The degree matrix

$$D = \begin{pmatrix} 17.35 & 0 & 0 & 0 & 0 \\ 0 & 7.22 & 0 & 0 & 0 \\ 0 & 0 & 6.38 & 0 & 0 \\ 0 & 0 & 0 & 18.99 & 0 \\ 0 & 0 & 0 & 0 & 11.96 \end{pmatrix}$$

gives the Laplacian as $L = D - A$:

$$L = \begin{pmatrix} 17.35 & 0 & 0 & -16.00 & -1.35 \\ 0 & 7.22 & 0 & -1.78 & -5.44 \\ 0 & 0 & 6.38 & -1.21 & -5.17 \\ -16.00 & -1.78 & -1.21 & 18.99 & 0 \\ -1.35 & -5.44 & -5.17 & 0 & 11.96 \end{pmatrix}$$

The eigenvalues of L, computed via `scipy.linalg.eigh`, are:

$$\lambda_1 = 0.00, \qquad \boxed{\lambda_2 = 3.54}, \qquad \lambda_3 = 7.22, \qquad \lambda_4 = 13.27, \qquad \lambda_5 = 28.99$$

Since $\lambda_2 > 0$, the graph is connected. The **Fiedler value** $\lambda_2 = 3.54$ serves as our baseline connectivity measure.

3.4.2 Laplacian Matrix After Adding AP$_C$

Adding AP$_C$ expands the graph to 6 nodes.
The adjacency matrix becomes:

$$A = \begin{pmatrix} 0 & 0 & 0 & 16.00 & 1.35 & 4.00 \\ 0 & 0 & 0 & 1.78 & 5.44 & 4.00 \\ 0 & 0 & 0 & 1.21 & 5.17 & 1.32 \\ 16.00 & 1.78 & 1.21 & 0 & 0 & 0 \\ 1.35 & 5.44 & 5.17 & 0 & 0 & 0 \\ 4.00 & 4.00 & 1.32 & 0 & 0 & 0 \end{pmatrix}$$

The degree matrix becomes:

$$D = \begin{pmatrix} 21.35 & 0 & 0 & 0 & 0 & 0 \\ 0 & 11.22 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7.70 & 0 & 0 & 0 \\ 0 & 0 & 0 & 18.99 & 0 & 0 \\ 0 & 0 & 0 & 0 & 11.96 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9.32 \end{pmatrix}$$

the new Laplacian L_{new} gains a row and column for AP$_C$ and becomes:

$$L_{\text{new}} = \begin{pmatrix} 21.35 & 0 & 0 & -16.00 & -1.35 & -4.00 \\ 0 & 11.22 & 0 & -1.78 & -5.44 & -4.00 \\ 0 & 0 & 7.70 & -1.21 & -5.17 & -1.32 \\ -16.00 & -1.78 & -1.21 & 18.99 & 0 & 0 \\ -1.35 & -5.44 & -5.17 & 0 & 11.96 & 0 \\ -4.00 & -4.00 & -1.32 & 0 & 0 & 9.32 \end{pmatrix}$$

The new Fiedler value is $\lambda_2^{\text{new}} = 5.09$, a **44% improvement** over the baseline.

3.5 Results

Access Points	Fiedler value λ_2	Change
2 APs baseline	3.54	—
3 APs (adding AP$_C$)	5.09	$+44\%$

Table 3.2: Connectivity comparison before and after adding AP$_C$.

Since AP$_C$ at $(50,\ 0)$ maximises the **Fiedler value** among all candidate positions tested, we can conclude that adding a third access point AP$_C$ significantly improves network connectivity.

3.6 General Application

To add any n number of APs to an existing network of buildings and APs, a general strategy would be as follows:
1. Construct a matrix directly from the geometric distances between APs and buildings in the existing network, i.e. given $AP_1, AP_2, \ldots, AP_m$ as access points and an amount of buildings/halls as $B_1, B_2, \ldots, B_n$, we would obtain an $n \times m$ matrix as shown below:

$$D = \begin{pmatrix} d_{11} & d_{12} & d_{13} & \cdots & d_{1m} \\ d_{21} & d_{22} & d_{23} & \cdots & d_{2m} \\ d_{31} & d_{32} & d_{33} & \cdots & d_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & d_{n3} & \cdots & d_{nm} \end{pmatrix}$$

2. Obtain the coverage matrix by replacing each entry with its corresponding signal strength using the inverse square law; i.e. $S = S_0/d^2$:

$$S = \begin{pmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{1m} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{2m} \\ S_{31} & S_{32} & S_{33} & \cdots & S_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & S_{n3} & \cdots & S_{nm} \end{pmatrix}$$

3. Form the adjacency matrix A with the previously obtained signal strengths (with signal strengths replacing 1s from a traditional adjacency matrix, zeros everywhere else):

$$D_0 = \begin{pmatrix} S_{11} & 0 & 0 & \cdots & 0 \\ 0 & S_{22} & 0 & \cdots & 0 \\ 0 & 0 & S_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & S_{nn} \end{pmatrix}$$

4. Sum each row of the adjacency matrix to get diagonal values for the degree matrix D.
5. Calculate the Laplacian $L = D - A$ and obtain the eigenvalues. Repeat this process with a greedy algorithm selecting multiple locations for each AP, eventually choosing the one which gives the greatest overall increase in connectivity.

Chapter 4

Conclusion

This project demonstrates how concepts from graph theory and linear algebra can be effectively applied to solve real-world network optimization problems, specifically WiFi coverage on the University of Ghana campus. By modelling buildings and access points as a weighted bipartite graph and incorporating signal strength through the inverse square law, we were able to translate a physical connectivity issue into a mathematical framework.

Using adjacency, degree, and Laplacian matrices, we analyzed the structural properties of the network and evaluated connectivity through spectral methods. In particular, the **Fiedler value** (the second smallest eigenvalue of the Laplacian) served as a reliable metric for measuring overall network connectivity. Our results showed that the initial network was connected, but not optimally so.

The introduction of the new access point (AP_C) and the application of a greedy optimization strategy led to a measurable improvement in connectivity. Specifically, the **Fiedler value** increased from 3.54 to 5.09, representing a **44%** enhancement. This confirms that the strategic placement of access points, guided by mathematical analysis, can significantly strengthen network performance.

Chapter 5

Summary

Throughout this research we have successfully been able to demonstrate how concepts in graph theory and linear algebra can be combined and applied to a network connectivity problem. Taking into consideration infrastructural, monetary, as well as other factors, we acknowledge that this may require further enquiry and research on our part, nonetheless, we believe that this method has enormous potential and can be applied to areas even beyond university campuses. Overall, this study highlights the power of interdisciplinary approaches; that is, combining mathematical theory with practical data to address infrastructure challenges. The methodology developed here is scalable and can be extended to larger and more complex campus networks, offering a systematic way to improve WiFi accessibility and user experience.

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